SYLLABUS

1. Data about the program of study

1.1	Institution	Technical University of Cluj-Napoca
1.2	Faculty	Faculty of Electrical Engineering
1.3	Department	Electrotechnics and Measurements
1.4	Field of study	Electrical Engineering
1.5	Cycle of study	Bachelor of Science
1.6	Program of study/ Qualification	Electrical System Cluj-Napoca in English language
1.7	Form of education	Full time
1.8	Subject code	17.00

2. Data about the subject

2.1	Subject name				Numerical Methods	
2.2	Course responsible/ lecturer				*Assoc.Prof.Eng. Levente CZUMBIL, PhD*	
2.3	Teachers in charge of Seminars/ Laboratory/ Project				*Assoc.Prof.Eng. Levente CZUMBIL, PhD*	
2.4 Year of study		II	2.5 Semester	1	2.6 Type of assessment *(E – exam, C – colloquium, V – verification)*	E
2.7 Subject category		*DF – fundamental, DD – in the field, DS – specialty, DC – complementary*				DF
		DI – compulsory, DO – elective, Dfac – optional				DI

3. Estimated total time

3.1 Number of hours per week:	4	of which	3.2 Course	2	3.3 Seminar	0	3.3 Laboratory	2	3.3 Project	0
3.2 Total hours per semester	56	of which	3.5 Course	28	3.6 Seminar	0	3.6 Laboratory	28	3.6 Project	0

3.7 Individual study:	
(a) Manual, lecture material and notes, bibliography	22
(b) Supplementary study in the library, online and in the field	5
(c) Preparation for seminars/laboratory works, homework, reports, portfolios, essays	10
(d) Tutoring	4
(e) Exams and tests	3
(f) Other activities	0

3.8 Total hours of individual study *[sum (3.7(a) to 3.7(f))]*	44
3.9 Total hours per semester *[sum of 3.4 and 3.8]*	100
3.10 Number of credit points	4.0

4. Prerequisites (where applicable)

4.1	Curriculum	▪ Mathematical Analysis, Linear Algebra, Special Mathematics, ▪ Computer Programming, ▪ Electric Circuits Theory, Physics
4.2	Competences	▪ Use of software applications, especially Microsoft Office ▪ Ability to operate with fundamental concepts in computer science ▪ Implementation of algorithms in programming languages ▪ Calculus: to solve analytically derivatives and/or integrals ▪ Solving direct current and alternating current electrical circuits

5. Requirements (where appropriate)

5.1	For the course	Video projector
5.2	For the applications	Computer Lab with Mathcad Prime software

6. Specific competences

Professional competences	▪ Ability to identify, formulate and solve engineering problems in a systemic approach. ▪ Description of the basic concepts and theories using the of appropriate mathematical approach and methods for the field of electrical engineering. ▪ Explain and interpret specific phenomena presented at field and/or specialty disciplines, using the fundamental knowledge of mathematics and physics. ▪ Application of scientific rules and methods to solve electrical engineering specific problems. ▪ To transform electrical engineering problems into numerical evaluation algorithms. ▪ Ability to use dedicated numerical evaluation software applications in order to solve specific electrical engineering problems. ▪ Appreciation of the precision, advantages and disadvantages of some numerical methods and procedures in the field of electrical engineering, as well as the level of scientific documentation of projects and the consistency of programs using scientific methods and mathematical techniques. ▪ To evaluate and analyze the results obtained from CAD software applications used to solve problems in the field of electrical engineering.
Cross competences	▪ Ability to work in groups/teams, to communicate effectively and to understand professional responsibilities. ▪ Flexibility in approaching and using different software applications and technologies in practice, in the assumed area of competence.

7. Expected learning outcomes

Knowledge	▪ The student/graduate identifies and describes basic concepts, principles and methods in mathematics, computer science, physics and economics. ▪ The student/graduate explains and interprets theoretical and experimental results from mathematics, computer science, physics and economics.
Abilities	▪ The student/graduate operates with basic concepts, principles and methods from fundamental disciplines. ▪ The student/graduate solves mathematics, physics and chemistry problems with applicability in engineering and validates the solution obtained. ▪ The student/graduate performs engineering and economic calculations of medium complexity and associates them with electrical or computer-aided design-specific graphic representations. ▪ The student/graduate applies criteria and evaluation methods for the identification, modeling, experimentation, analysis and qualitative and quantitative assessment of phenomena and processes specific to the fundamental field, including using digital technologies. ▪ The student/graduate acquires and processes data, interprets theoretical and experimental results.
Responsibility and autonomy	▪ The student/graduate practices logical reasoning, evaluation, and self-assessment in decision-making. ▪ The student/graduate is engaged in lifelong learning to acquire and implement knowledge as needed, using appropriate learning strategies. ▪ The student/graduate communicates effectively about engineering activities with a wide range of audiences. ▪ The student/graduate promotes dialogue, cooperation, respect for others and interculturality. ▪ The student/graduate works effectively as a team member or team leader.

8. Discipline objectives (based on specific competencies acquired)

8.1	General objective	▪ Development of skills in the field of Numerical Methods to support professional training in electrical engineering. ▪ The course aims to introduce students to both classical and new, advanced numerical methods that can be applied to solve problems in the field of electrical engineering, power and/or electrical systems, in conjunction with the use of appropriate computational techniques.
8.2	Specific objectives	▪ The students will learn how to apply numerical computational methods to solve specific problems in electrical engineering and power systems, ensuring the development of rigorous numerical logic and an algorithmic way of thinking. ▪ Several numerical methods will be presented for various problems that will be addressed at lectures and laboratory applications highlighting the importance of performing an analysis on algorithm complexity, their numerical stability, as well as their implementation on a computer. ▪ To learn to use advanced techniques to implement algorithms in numerical computation software. ▪ To choose the appropriate numerical methods and techniques for any problem that electrical engineers could meet in practice.

9. Contents

9.1. Course (Lectures)		Number of hours	Teaching methods	Additional remarks
1	**Chapter 1. The use of numerical methods in Electrical Engineering applications** Introduction. Purpose of the course. The evolution of numerical methods and computational techniques. Evaluation of errors that could appear in solving numerical problems. How to properly express errors. Sources and types of errors. Error propagation. Evaluation of mathematical functions used in electrical engineering. Computing analytical functions, polynomial functions, function values using successive approximations and series expansions. Use of complex numbers in electrical engineering applications. Practical rules. Reference materials.	2	Power Point Presentation Interactive Discussions Demonstrations, Explanations of Algorithms on the board Demonstrations of Convergence, Stability of Methods on the board/slides	Onsite sessions according to IE schedule
2	**Chapter 2. Numerical Methods applied to solve Algebraic and Transcendental Equations** Presented numerical methods: Bisection Method, Newton-Raphson Method, Secant Method, Halley's Method. Applications that will be discussed during the lecture: Solving nonlinear resistive DC circuits;	2		

	Determining the optimal pathway of an overhead power line; Solving the equilibrium equation for electric force versus electrodynamic force in case of electrostatic powder coating applications.			
3 4	**Chapter 3. Numerical Methods used to solve Systems of Equations** Matrix Algebra Fundamentals: Matrix inversion and factorization, Matrices and circuit graphs, Vector and Matrix Norms. Direct and Iterative Methods for Eigenvalue Computations. Study of conditioning and solving ill-posed systems of equations. Numerical Solutions of Linear Systems of Equations: Gaussian Elimination, LU Factorization, Jacobi's Successive Approximations Method. Applications that will be discussed during the lectures: Analysis of linear and nonlinear electrical circuits under steady-state, harmonic, or transient conditions; Stability analysis of electrical systems under small disturbances; Procedures for solving DC and AC electrical circuits using matrix methods and condition number analysis.	4		
5 6	**Chapter 4. Numerical Methods for Numerically given Functions Approximation and Interpolation** Least Squares Method to determine approximation function coefficients, Function Approximations with Taylor Series, with Fourier Series – Harmonic Analysis. Polynomial Interpolation: Lagrange Interpolation, Newton Interpolation with Divided Differences, Spline Functions. Applications that will be discussed during the lectures: Distribution boards placement; Load curves processing through interpolation; Numerical approximation of the no-load characteristic of a synchronous generator.	4		
7 8	**Chapter 5. Numerical Methods to Evaluate Definite Integrals** Newton-Cotes Quadraure Formulas: Trapezoidal Rule; Generalized Trapezoidal Rule; Simpson's Rule; Generalized Simpson's Rule. Gauss Quadrature Formulas. Applications that will be discussed during the lectures: Evaluation of Overvoltages induced by lightning on overhead power lines; Electricity Consumption evaluation based on power load curves; Counting electrons through a conductor in case of a nesinusoidal current.	4		

9 10	**Chapter 6. Numerical Computation of Derivatives** Numerical Differentiation Based on Interpolation Polynomials: Derivation of Lagrange Interpolation Polynom; Derivation of Newton Interpolation Polynom. Differentiation Formulas using Taylor Series Expansions. Applications that will be discussed during the lectures: Determining the distribution of electric charges; Calculating the intensity of electromagnetic fields.	4		
11 12	**Chapter 7. Numerical Methods for Solving Differential Equations and Systems of Equations** Numerical Methods for solving Ordinary Differential Equations: Taylor Series Expansion Method; Euler's Method; Modified Euler Methods; Runge-Kutta Methods. Solving Systems of Ordinary Differential Equations. Solving Higher Order Differential Equations. Solving Partial Differential Equations: Parabolic Equations; Hyperbolic Equations; Elliptic Equations. Applications that will be discussed during the lectures: Solving electrical circuits under transient conditions; Analyzing the behavior of surge arresters.	4		
13	**Chapter 8. Fundamentals of the Finite Difference Methos and of the Finite Element Method** Numerical Methods for Solving the Differential Mathematical Model of Electromagnetic Fields: Solving Laplace's Equation using the Finite Difference Method. Tools that utilize Finite Element Method FEM and Finite Difference Methos FDM for numerical modeling and simulation. Applications that will be discussed during the lectures: Electromagnetic field analysis and modeling; Solving complex boundary value problems in electrical and energy systems.	2		
14	**Chapter 9. Fundamentals of Artificial Intelligence Techniques** Introduction to Artificial Intelligence Techniques: Fuzzy Logic; Neural Networks; Genetic Algorithms. Numerical. Applications of Artificial Intelligence Techniques in Electrical and Power Engineering.	2		

Bibliography

[1] D.D. Micu, A. Ceclan, Metode Numerice. Aplicații în ingineria electrică, Ed. Mediamira, 2007.
[2] D. Ioan, ş.a. Metode numerice în ingineria electrică. Ed. Matrix Rom Bucureşti, 2005.
[3] Ş. Kilyeni, Metode numerice. Aplicații în energetică, Ed. Orizonturi Universitare Timişoara, 2006.
[4] D.D. Micu, Metode numerice în studiul interferenţelor electromagnetice, Ed. Mediamira, 2004.
[5] S.C. Chapra, R.P. Canale, Numerical Methods for Engineers, Ed. McGraw-Hill, New York, USA, 2015.

[6] E. F. James, An Introduction to Numerical Methods and Analysis, John Wiley & Sons, NY, 2001.
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[8] J.M. Jin, Theory and computation of electromagnetic fields, Ed. Wiley, IEEE Press, 2010.
[9] G. Lehner, Electromagnetic Field Theory for Engineers and Physicists, Ed. Springer, 2009.
[10] R. Burden, J.D. Faires, Numerical Analysis, Ed. Brooks/Cole Publishing Company, 2001.
[11] Rus, I. Crăciun, Modelare matematică, Ed. Transilvania Press, Cluj-Napoca, 2000.
[12] H. Shang-Xu, Applied Numerical Computation Methods, Ed. ZJU, MIT – USA, 2011.

9.2. Applications - Laboratory		Number of hours	Teaching methods	Additional remarks
1	**Laboratory 1. General Overview of Mathcad Prime. Introductory Applications in Electrical Engineering** Data types used in electrical engineering. Implementing numerical algorithms in Mathcad. Programming palette and symbolic calculation. Solving simple electrical circuit theory applications.	2	The laboratory sessions (Practical Applications in the MathCad Program) are based on an interactive partnership between the instructor and the student.	Onsite sessions according to IE schedule
2 3	**Laboratory 2+3. Numerical methods for solving algebraic and transcendental equations.** Mathcad functions that can be used to solve algebraic and transcendental equations numerically. Applications in electrical engineering implemented in MathCad using: The Bisection Method; Newton's Method Secant Method; Halley's Method. (The following applications will be discussed: The effect of starting electric machines. Establishing the optimal path-way of a power line)	4		
4 5 6	**Laboratory 4+5+6. Numerical methods for solving Equation Systems** Mathcad functions that can be used to solve systems of equations numerically. Operations with vectors and matrices. Calculation of eigenvalues and vectors. Conditioning number. Applications in electrical engineering implemented in MathCad using: Gauss method; Jacobi method. The following applications will be discussed: Matrix solution of d.c./a.c. electrical circuits. Free-state analysis of a linear electrical circuit. Analysis of linear circuits in steady state.	6		
7	**Laboratory 7+8+9. Approximation and Interpolation of Numerically given Functions.** Mathcad functions that can be used for approximations and interpolations. Operations with	6		

8 9	vectors and matrices. Calculation of eigenvalues and vectors. Conditioning number. Applications in electrical engineering implemented in MathCad using: spline polynomial interpolation; Lagrange interpolation; Newton interpolation with divided differences The following applications will be discussed: Analytical approximation of tabular functions used in the sizing of electrical installations - placement of distribution boards. Testing of overhead power line insulators.			
10	**Laboratory 10. Numerical Calculation of Definite Integrals.** Using the integration operator in Mathcad for the analytical evaluation of indefinite integrals. Implementing formulas for numerical evaluation of definite integrals: approximation based on the area evaluation: rectangles formula; generalized trapezoid formula; generalized Simpson's formula. Numerical evaluation of double integrals. The following applications will be discussed: Determining energy consumption based on the daily load curve; Determining the total electric flux given by an electric charge through a Gaussian surface.	2		
11	**Laboratory 11. Approximate Evaluation of Derivatives.** Using the derivative operator in Mathcad for the analytical evaluation of the derivative of a mathematical function. Implementing calculation formulas for the numerical evaluation of the value of the derivatives of mathematically and/or numerically defined functions: formulas obtained by deriving Lagrange interpolation polynomials. The following applications will be discussed: Determining electric currents through three-phase power factor compensation capacitors; Modeling the movement of a high-speed electric train.	2		
12 13	**Laboratory 12+13. Numerical solution of equations and systems of differential equations.** Mathcad functions that can be used to numerically solve differential equations and/or systems of differential equations Applications in electrical engineering implemented in MathCad using: Taylor series expansion method; Euler's method; Runge-Kutta methods. The following applications will be discussed: Numerical analysis of	4		

	electrical circuits in transient regime. Use of differential mathematical models in automation and electrical drive applications.			
14	**Laboratory 14. Review Applications.** Review of the main solving methods and numerical solving tools available in the Mathcad utility, which were studied throughout the semester.	2		

Bibliography

[1] D.D. Micu, A. Ceclan, L. Czumbil, D. Csala, *Metode Numerice. Lucrări practice*, Ed. Mediamira, 2010.

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[3] M. Sadiku, *Numerical Techniques in Electromagnetics*, CRC Press, 2000

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[6] D.D. Micu, A. Cziker, *Aplicaţii ale metodelor numerice în electrotehnică*, Ed. Casa Cărţii de Ştiinţă, 2002.

[7] A. Ceclan, D.D. Micu L. Czumbil, H. Andrei, M. Găiceanu, M. Stănculescu, P.C. Andrei: „*Ill-Posed Inverse Problems in Electrical Engineering Applications*", Numerical Methods for Energy Applications, Ed. Springer, ISBN: 978-3-030-62190-2, Ch. 9, pp. 235-258 2021.

10. Alignment of course content with expectations of the epistemic community, professional associations, and representative employers in the field

The competencies acquired will be essential for employees working in the fields of electrical engineering, energy engineering, and applied engineering sciences. The course content aligns with similar programs offered in other faculties of electrical engineering, both within the Technical University and in other academic centers across the country and abroad. For a better adaptation of he course content to the demands of the labor market, meetings were held with representatives from companies in the field.

Curricular compatibility of the content of the discipline with similar study programs at technical universities abroad:
https://ocw.mit.edu/courses/18-335j-introduction-to-numerical-methods-spring-2019/
http://www.mcgill.ca/study/2014-2015/courses/ecse-443

11. Assessment

Activity type	11.1 Assessment criteria	11.2 Assessment methods	11.3 Weight in the final grade (%)
11.4 Course	Knowledge of principles / creative and critical thinking in the application of knowledge. Solving a test that covers all the topics covered in the lectures.	Written Test	40%
11.5 Laboratory	Based on the practical knowledge gained	Practical test on computer	60%

11.6 Minimum standard of performance:

Understanding basic concepts and terminology; Problem solving

Requirements to enter the exam: **Minimum 5 attendances at Lectures** and to perform of **all the laboratory activities** according to ETCS regulations.

Date of completion	**Lecturers**	**Title/ Surname/ Name:**	**Signature**
02.09.2025	Course	*Assoc.Prof.Eng. Levente CZUMBIL, PhD*	
	Applications Laboratory	*Assoc.Prof.Eng. Levente CZUMBIL, PhD*	

Date of approval in the ETHM Department Council September 2025	**Head of Department:** Prof. Eng. MICU Dan Doru, PhD
Date of approval in the Faculty of Electrical Engineering Council September 2025	**Dean:** Assoc. Prof. Eng. CZIKER Andrei, PhD